SEASONALITY AND THE CANADIAN FINANCIAL FLOW STATISTICS: BALANCING THE SEASONALLY ADJUSTED SERIES

by

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As work-in progress, any conclusions contained herein, are those of the author and do not represent the view of Industry Measures and Analysis Division or of Statistics Canada.

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SEASONALITY AND THE CANADIAN FINANCIAL FLOW STATISTICS: BALANCING THE SEASONALLY ADJUSTED SERIES

1. Introduction

The issue of seasonality in the Canadian financial flow statistics was last studied in the mid-seventies. That study concluded that "it would appear that seasonal behaviour is poorly defined in the Canadian financial flow series. As a consequence, the seasonally adjusted series are not very reliable and actually could be misleading."[1] This conclusion was based on three facts. First, the financial data contain relatively large irregular components which tend to hide the true seasonal patterns. Second, the two-way constraints (demand equals supply for each category of financial securities and net savings are equal to the net change in financial assets minus the net change in financial liabilities for each sector of the economy) complicates the seasonal adjustment process as the standard version of X-11[2] does not preserve the identity of an aggregate series and its components. Third, the financial flow series were at that time shorter, and, the seasonal component would not have been easily identifiable.

In seasonally adjusting the Canadian financial flow statistics, two main considerations must be studied: first, the issue of seasonally adjusting individual financial flow series, and second, the balancing constraints of 1) financial assets and liabilities for each sector of the economy and 2) demand and supply for each category of financial securities.

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[1]Seasonality in the Financial Flow Accounts, Financial Flow Accounts, Statistics Canada, Cat. No. 13-002, fourth quarter, 1975.

[2]The X-11-ARIMA Seasonal Adjustment Method, Estela B. Dagum, Statistics Canada, Cat. no. 12-564E Occasional.

In the latter case, it must be noted that the balancing constraints in the financial flow table do not hold automatically after the seasonal adjustment of individual series.

A previous paper[3] dealt with the feasability of seasonally adjusting individual financial flow series. That study has been performed on a small number of financial flow series which also have the balancing constraints. This subset of Canadian financial flow statistics also allows us to study the feasibility of balancing the financial flow table (see Appendix A).

The main conclusion of this earlier study is that it is quite feasible to seasonally adjust individual financial flow series.

From that point, balancing the seasonally adjusted individual financial flows series was considered in order to make a consistent seasonally adjusted financial flow table.

We study mainly two approaches. The first method is based on time domain and is currently used in the United States. The second method is based on frequency domain and is an extension to a method developed by D. Rhoades and N. Elhawary-Rivet[4] to adjust sub-annual data to an annual benchmark. Both approaches are however using a least square type of minimization.

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[3]Seasonality and the Canadian Financial Flow Statistics: Report on a Study in Progress, James A. Brox and Jacques Taillon, Econometric Analysis Division, Working Paper 86:01, Statistics Canada, January, 1986

[4]Adjusting time Series to Benchmarks using Frequency - Domain Properties of the Data, Darryl Rhoades and Nawal Elhawary-Rivet, Statistics Canada, forthcoming 1986

In section 3 of this study, the background motivation for the study is outlined. Section 4 presents the different types of constraints on the financial flow table. Sections 5 and 6 explain the approaches using the frequency and time domain respectively in order to balance the seasonally adjusted financial flow table.

2. Summary

To follow

3. The Background Motivation

Much of the motivation for this study followed from the need for financial data in conducting current analysis, and from an attempt to model the economic behaviour[5] recorded in the financial flow data matrix (see Appendix A). With respect to current analysis, the lack of seasonally adjusted data results in awkward expressions that try to put the most recent figures in perspective. For example:

> "Demand for funds by the household sector through conventional market instruments totalled an unadjusted $5.4 billion in the fourth quarter (of 1984), which is two and a half times borrowings in the third quarter but represents a slight decline compared with the fourth quarter of 1983."[6]

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[5] See James A. Brox and Wendy A. Maclean, An Econometric Model of the Canadian Financial Flow Matrix, (forthcoming, 1986).

[6] Quarterly Economic Summary, April 1985, Statistics Canada, Catalogue no. 13-006.

In addition to being awkward, the statement is not very informative as to whether or not the increase in household demand for funds is significant in terms of the evolution of the business cycle. The existence of seasonally adjusted data would result in more concise and informative statements.

The modelling exercise was designed to create a model of the financial sector that could subsequently be interacted with a standard model of the real sectors of the Canadian economy. However, most current macro-econometric models of the Canadian economy have been constructed using seasonally adjusted quarterly data. Since the financial flow data are available only in raw form, the union of the financial model and the standard macroeconometric model would require that the seasonality present in the financial data contribute a relatively insignificant proportion of the total variation.

The functional form of the behavioural equations in the financial flow model included seasonal dummy variables to account directly for the purely seasonal variation. These dummy variables were viewed as portfolio shift effects and thus were constrained to sum to zero across the entire set of equations for each sector of the model. The empirical results revealed that in almost every sector of the model, strongly significant seasonal effects were present. Thus the plans to join the financial flow model with a model of the real sector were delayed pending the results of a formal study of the problems of seasonally adjusting the matrix.

A second alternative would have been to develop a model of the real sector based upon the raw data.[7] This approach was rejected because one of the major purposes of such models is to aid in current economic analysis, and strong seasonal variations tend to mask underlying economic trends. Thus it was felt that a study of the problems involved in seasonally adjusting the entire financial flow matrix would produce more substantial long-run benefits.

4. Constraints on the Financial Flows Table

The financial flow table is a complex set of time series which are related to each other. Each sector is divided into assets and liabilities which are also subdivided into 21 categories such as currency and bank deposits, bank loans, government of Canada bonds and so forth (lines 2100 to 3610 in Appendix A). There are 14 sectors such as persons and unincorporated Business, Chartered Banks and so forth.

An important aspect of the financial flow table is the attempt to reconcile the total financial flow figures in terms of the net financial investment series (line 2000) and the corresponding information (net lending or borrowing, line 1900) from the national income and expenditure accounts. The difference between these two series is given by the discrepancy variable (line 4000).

[7] One obvious course would have been to update the original version of SCQUEM (Statistics Canada Quarterly Econometric Model) which was estimated using raw data.

Depending on which sector, the discrepancy value can be quite large. However, it has been noticed that there is a fair amount of seasonality in the discrepancy variables. We then expect to decrease the discrepancy values between the net financial investment and the net lending or borrowing in seasonally adjusting the financial flow table.

In the next sections, the discrepancy variable has not been included but the procedure would be the same by replacing the seasonally adjusted net financial investment variable by the seasonally adjusted net lending or borrowing variable.

4.1 Notation

The financial flow table can be written as a vector using the following notation. Each individual financial flow series is classified by category (i) and by sector (j).

Let x_{ijt} the financial flow series for category i
and sector j at time t.
where i = 1,2, -- C (total number of categories)
j = 1,2, -- S (total number of sectors)
t = 1,2, -- N (total number of quarters
in the time series).

y_{it} the total financial flow series for category i at time t

z_{jt} the net financial investment (asset minus liabilities)
for sector j at time t.

Let

$$X = \begin{bmatrix} (x_{ijt}) \\ (y_{it}) \\ (z_{it}) \end{bmatrix}_{(LxN)}$$

where $(x_{ijt})_{(2xCxSxN)x1}$

$(y_{it})_{(2xCxN)x1}$

$(z_{it})_{(SxN)x1}$ are vectors. There are 2xC categories in the financial flows matrix and

$L = [(2xCxS)+(2xC)+S]$

4.2 Constraints on the Raw Financial Flow Table

There are three types of constraints that we have to take into account when the financial flow table is seasonally adjusted. These constraints are related to the three indices of x_{ijt}.

a) Category Constraints

The sum of all sectors for a given asset or liability category is equal to the total for the corresponding asset or liability category:

$$\sum_{j=1}^{S} x_{ijt} = y_{it} \quad \forall t, \forall i$$

$$\sum_{j=1}^{S} x_{i+C,jt} = y_{i+C,t} \quad \forall t, \forall i$$

The total (sum) for a given asset category is equal to the total (sum) for the corresponding liability category:

$$y_{it} = y_{i+C,t} \quad \forall t, \forall i$$

b) Sector Constraints

The difference between total assets and total liabilities for a given sector is equal to the net financial investment.

$$\sum_{i=1}^{C} x_{ijt} - \sum_{i=1}^{C} x_{i+C,jt} = z_{jt} \quad \forall t, \forall j$$

c) Time Constraints

The sum of the four quarters in a given year for a given sector and category must be equal to the yearly raw total for the sector and category.

4.3 Constraints on the Seasonally Adjusted Financial Flow Table

After each individual financial flow series has been seasonally adjusted, we have to assure that the category, sector and time constraints still hold. In the next two sections, a technique using the frequency and time domains respectively, are outlined.

5.0 Methodology to Balance the Seasonally Adjusted Financial Flow Table in Frequency Domain

5.1 The Basic Approach

In order to balance the seasonally adjusted financial flows table, the basic idea is to modify as little as possible each individual series in order to meet the balancing constraints.

Several financial flow series, however, have no seasonality identified. In these cases, the balancing procedure will leave unchanged those series without seasonality. Only financial flow series with identifiable seasonality will be, in fact, seasonally adjusted and then, eventually modified in the balancing procedure.

These modifications must alter very slightly some function of the data. Quite frequently, this function is a least squares type of function using the time domain data. A new approach outlined in these pages deals with the frequency domain data.

Indeed, we want in the balancing procedure, to modify as little as possible the periodogram (or spectral density) of each individual financial flows series which was seasonally adjusted.

5.2 Balancing and Time Constraints in a Matrix Notation

The vector X will consist of all individual seasonally adjusted series before the balancing act.

a) Balancing Constraints

The balancing constraints can be written as such

$$QX = 0$$

where Q is a matrix $(N_C \times N) \times (L \times N)$ of 1 and 0 values taking into account the category and sector constraints

$N_C = 3C + S$ is the number of linear balancing constraints.

b) Time Constraints

The time constraints can be written as

$$BX = X^A$$

where B is a matrix $(Lx\frac{N}{4})$ x (LxN) of 1 and 0 values taking into account the time constraints.

X^A is the vector $(Lx\frac{N}{4})$ x 1 of raw (not seasonally adjusted) annual financial flows

c) Balancing and Time Constraints

The balancing and time constraints can then be written jointly as

$$\begin{bmatrix} B \\ Q \end{bmatrix} \quad X \quad = \begin{bmatrix} X^A \\ 0 \end{bmatrix}$$

A detailed description of B and Q matrices can be found in Appendix B.

5.3 The Minimization Equation

In order to calculate the Fourier Transform (for the periodogram) of each series in the financial flow table, we must build a matrix F of Fourier coefficients which is NxN (see Appendix C). We repeat the matrix F on the diagonal to build the matrix $\tilde{F}$ such that $\tilde{F}X$ contains the Fourier transforms of X.

If X^B represents the balanced data, then we want $\tilde{F}X^B$ as close as possible to $\tilde{F}X$, subject to the balancing and time constraints.

To achieve this, we would like to minimize the least squares function

$$(X^B - X)\ \tilde{F}'\tilde{F}\ (X^B - X)$$

In pratice however, two transformations of vectors X and X^B must be done in order to calculate the Fourier transforms and the minimization: stationarization and standardization. (see Appendices D and E respectively).

After the minimization is performed, (see Appendix F) we must reverse the standardization and the stationarization operations to find the final seasonally adjusted balanced financial flow data.

5.4 Stationarization and Standardization of Seasonally Adjusted Financial Flows Series

5.4.1 Stationarization

Since the frequency domain method is based on estimating the Fourier transform of the seasonally adjusted series, then it is important that the seasonally adjusted series be stationary with respect to the mean and the variance. The balanced seasonally adjusted series is obtained such that the sum of squared differences between the Fourier transforms of the seasonally adjusted and the balanced seasonally adjusted series is minimized, subject to the constraints.

In order to stationarize the seasonally adjusted financial flow series, the technique developed in the step adjustment procedure for

an individual series will be applied to each financial flow series.This stationarization technique can be outlined in the following manner:

a) taking annual difference
b) dividing by the annual total of the seasonally adjusted series
c) minimizing over high frequencies

See Appendix D. Steps a and b affect the constraints matrices B and Q since the matrix X is modified.

5.4.2 Standardization

A further step must be taken in order to be able to compare adequately periodograms of different series which are of different scales. Indeed, in the minimization procedure, several periodograms will be modified according to a sum of squared differences. These periodograms must have more or less the same magnitude; otherwise, the minimization process would be distorted putting to much emphasize on the periodograms of large magnitude.

In order to take into account the relationship between periodograms in the minimization process, a standardization of the series is performed. The standardization comes after the seasonally adjusted series were stationarized. The normal standardization is used. See Appendix E.

6.0 Methodology to Balance the Seasonally Adjusted Financial Flows Table in Time Domain

6.1 The Basic Approach

As stated in section 5.1, in order to balance the seasonally adjusted financial flows table, we would like to modify as little as possible each individual series in order to meet the balancing constraints.

A method using the time domain has been in force in the USA for more than twenty years. It balances one quarterly financial flow table at a time and the time constraints are considered within this approach after the four quarters of a calendar year have been balanced.

Weights are given to each financial flows series in order to assure which series have to stay unchanged, which ones could be modified slightly or more than slightly.

6.2 Balancing Constraints in a Matrix Notation

The balancing constraints are the same as in the frequency domain approach. The only difference is that the matrix Q is N_CxL instead of (N_CxN) x (LxN) since we consider in the time domain approach only one quarter at a time instead of a whole time series.

A special remark must be addressed about the time domain constraints. We still want the sum of a given seasonally adjusted financial flows series of the four quarters in a given year to be equal to the yearly raw total for the same series. The time constraints would be similar to the ones in the

frequency domain approach and would be used in the balance calculations only when the four quarters of a calendar year of balanced seasonally adjusted data are available.

6.3 The Weighting Metrix

Several weighting matrices can be used to test the time domain approach. A weighting matrix is built with weights for each individual series where the total sector or total category weight is the sum of the corresponding weights of the individual series. For a given quarter, let's define;

x_{ij} is the seasonally adjusted financial flow series for the ith category and the jth sector

$T = \{t \ni$ t is a quarter from a given period, for example the period from the first quarter of 1962 to the fourth quarter of 1984 inclusively$\}$

$T_3 = \{t \ni$ is a quarter from the third quarter of 1962 to the fourth quarter of 1984 inclusively$\}$

$F = (f_{ij})$ is the weighting matrix

6.3.1 Absolute Dollar Size of Second Difference of Each Unadjusted Financial Flows Series

$$\text{Let } f^*_{ij} = \frac{\sum_{t \in T_3} \left| (x_{ijt} - x_{ijt-1}) - (x_{ijt-1} - x_{ijt-2}) \right|}{\#T_3}$$

where $\#T_3$ is the cardinality of set T_3

The weights f_{ij} are a normalized version of f^*_{ij}

$$f_{ij} = \frac{f^*_{ij}}{\sum_{ij} f^*_{ij}} \quad \text{where} \quad f^*_{ij} \geq 0 \quad \text{and} \quad 0 \leq f_{ij} \leq 1$$

The US approach is using these weights.

6.3.2 Irregular Component

The weighting matrix can be built using the irregular component of the series as estimated by X-11-ARIMA. Indeed, this seasonal adjustment procedure calculates some quality statistics.[8] Six of these quality statistics are directly related to the irregular component of a series. The lowest each of these quality statistic values is, the better is the adjustment.

It would be however tedious to use each of these six quality statistics seperately as a weight. Instead, a weighted quality statistics based on the six quality statistics will be used. Using the weights which were developed for X-11-ARIMA and designating each of the six quality statistic values by M_i, $i = 1, - - , 6$, the weight f^*_{ij} would be

$$f^*_{ij} = (.24 \times M_1) + (.24 \times M_2) + (.14 \times M_3) + (.08 \times M_4) + (.16 \times M_5) + (.14 \times M_6)$$ and the final weight f_{ij} would be

[8] J. Lothian and M. Morry, "A Set of Quality Control Statistics for the X-11-ARIMA Seasonal Adjustment Procedure", Statistics Canada, Internal Paper, October, 1978

$$f_{ij} = \frac{f^*_{ij}}{\Sigma f^*_{ij}}$$

These weights could be fixed even if they could be calculated each quarter. The variation from one quarter to the following should be minimal since the financial flows series are in the most cases almost twenty-five years long.

6.3.3 Seasonal Frequency Bandwidth

The bandwidth seasonal frequency could be used as weights because a very narrow bandwidth should mean a good seasonal adjustment while a wide one should mean a more difficult evaluation and eventually a not so accurate seasonal adjustment.

6.4 The Minimization Equation

The minimization equation in time domain is similar to the one developed in frequency domain. In time domain, the F matrix is the weighting matrix instead of the Fourier coefficients matrix. If X^B represents the balanced seasonally adjusted data, then we want FX^B as close as possible to FX, subject to the balancing and time constraints. To achieve this, we would like to minimize the least square function

$$(X^B - X)' F' F (X^B - X)$$

APPENDIX A

Table 1-2 from Statistics Canada Publication:

Financial Flow Accounts, Cataloque no. 13-002

Table 1-2. Financial Flows Matrix, in millions of dollars, Third Quarter, 1985

Category	Sector/Secteur	I&II. Persons and Unincorp'd Business/ Particuliers et entreprises individuelles	III. Non-financial Private Corporations/ Sociétés privées non financières	IV. Non-financial Government Enterprises/ Entreprises publiques non financières	V. The Monetary Authorities / Les autorités monétaires	VI 1. Chartered Banks / Banques à charte	VI 2. Near-banks / Quasi-banques	VII. Insurance Companies and Pension Funds/ Sociétés d'assurance et régimes de pension	VIII. Other Private Financial Institutions/ Autres institutions financières privées
1100	Gross Saving	20133	10200	1082	2	1069	84	26	81
1101	Residual Error of Estimate, NIEA	...	...	...	...	...	...	...	...
1200	C.C.A. and Misc. Valuation Adjustments	3564	7237	1021	2	64	22	26	7
1400	Net Saving	16569	2963	61	...	1005	62	-	74
1500	Non-Financial Capital Acquisition	10594	11776	4234	4	131	-107	90	112
1501	Residual Error of Estimate, NIEA	...	...	...	...	...	...	...	...
1600	Gross Fixed Capital Formation	5078	11588	2995	4	131	-	12	105
1700	Value of Physical Change in Inventories	6115	1031	413	...	...	...	...	...
1800	Net Purchases of Existing Assets	-599	-841	886	...	-	-107	78	7
1900	Net Lending or Borrowing (1100-1500)	9539	-1576	-3212	-2	938	191	-64	-31
2000	Net Financial Investment (2100-3100)	10173	-2773	-1578	-11	613	218	-64	-31
2100	Net Increase, Financial Assets	16022	6269	-200	462	8336	2053	4111	4780
2210	Official International Reserves	...	...	...	112	...	...	...	...
2311	Currency and Bank Deposits	699	-76	-121	...	187	-229	259	298
2312	Deposits in Other Institutions	993	-24	-	...	...	82	17	245
2313	Foreign Currency and Deposits	262	-510	-100	...	-103	234	18	-222
2321	Consumer credit	-	-	...	...	1228	452	-1	157
2322	Trade Receivables	...	1428	273	...	...	...	92	24
2331	Bank loans	...	...	...	...	-599	...	...	...
2332	Other loans	...	180	18	810	...	25	-16	319
2340	Government of Canada Treasury Bills	3126	43	-13	-363	2650	-293	388	1142
2350	Finance and Other Short-term Paper	1404	-81	-241	-	2072	656	621	523
2410	Mortgages	83	93	-	...	1941	1632	890	21
2421	Government of Canada Bonds	1100	11	24	-180	-319	-387	401	235
	(of which Canada Saving Bonds)	-946	...	...	...	...	...	...	...
2422	Provincial Government Bonds	654	-12	20	...	-114	-92	654	-27
2423	Municipal Government Bonds	-365	-	1	...	-19	-2	70	126
2424	Other Canadian Bonds	-580	47	2	...	-194	28	298	-28
2430	Life Insurance and Pensions	4075	...	...	...	...	...	...	...
2512	Claims of Associated Corporations (1)	...	4554	-202	...	317	87	62	1681
2513	Government Enterprise Claims	...	...	91	-	...	...	...	...
2520	Stocks (1)	413	260	28	...	45	57	388	484
2530	Foreign Investments	-789	34	2	...	1318	24	132	260
2810	Other Financial Assets	4947	302	18	83	-72	-221	82	-438
3100	Net Increase, Liabilities and Share Capital	5849	9042	1378	473	7723	1835	4175	4811
3210	Official International Reserves	...	...	...	...	...	...	...	...
3311	Currency and Bank Deposits	...	...	...	-622	4045	...	...	...
3312	Deposits in Other Institutions	...	...	...	...	...	1569	...	-10
3313	Foreign Currency and Deposits	...	...	...	...	907	-40	...	...
3321	Consumer Credit	1838	...	...	...	...	...	...	...
3322	Trade Payables	-337	1082	136	...	...	...	-10	12
3331	Bank Loans	-107	-1600	40	...	...	-25	153	2291
3332	Other Loans	248	150	153	...	921	230	3	-358
3340	Treasury Bills Net of Unamort. Discount	...	...	...	...	...	...	...	...
3350	Finance and Other Short-term Paper	-8	5677	-6	...	...	-11	...	97
3410	Mortgages	4237	200	-2	...	...	-44	26	8
3421	Government of Canada Bonds	...	...	-1	...	...	...	...	...
3422	Provincial Government Bonds	...	...	795	...	...	...	...	...
3423	Municipal Government Bonds	...	...	-	...	...	...	...	...
3424	Other Canadian Bonds	-20	544	94	...	828	87	...	-350
3430	Life Insurance and Pensions	...	...	...	...	...	...	3398	102
3512	Claims of Associated Corporations (1)	...	1180	-	...	-	-240	13	421
3513	Government Enterprise Claims	...	...	275	894	...	...	...	...
3520	Stocks (1)	...	1509	-	...	107	71	18	1846
3530	Foreign Investments	...	...	...	...	...	...	...	...
3810	Other Liabilities	...	300	-106	201	915	238	-28	752
4000	Discrepancy (1900-2000)	-634	1197	-1634	9	325	-27	-	-

(1) For Footnotes see Table 7-1

Tableau 1-2. Matrice des flux financiers, en millions de dollars, troisième trimestre, 1985

IX. Public Financial Institutions/ Institutions financières publiques	X. Federal Government Administration publique fédérale	XI. Other Levels of Government and Hospitals/ Autres admin. publiques et hôpitaux	XII. Social Security Funds/ Caisses de sécurité sociale	XIII. Rest of the World/ Reste du monde	XIV. Residual Error Erreur résiduelle	Total Total	Catégorie
-139	-4870	2253	625	1236	-323	31459	1100 Epargne brute
...	...	...	...	...	-323	-323	1101 Erreur résiduelle d'estimation, CNRD
-	352	1724	...	...	...	14019	1200 Prov.pour cons.de cap.et ajust.divers
-139	-5222	529	625	1236	...	17763	1400 Epargne nette
-	808	2898	...	536	323	31459	1500 Acquisition de capital non financier
...	...	...	...	...	323	323	1501 Erreur résiduelle d'estimation, CNRD
-	748	2898	...	...	...	23557	1600 Formation brute de capital fixe
-	20	-	...	...	...	7579	1700 Valeur de la variation matérielle des stocks
-	40	-	...	536	...	-	1800 Achats nets d'actifs existants et intangibles
-139	-5678	-645	625	700	-646	-	1900 Prêt ou emprunt net (1100-1500)
137	-5569	-2162	625	422	-	-	2000 Investissement financier net (2100-3100)
1437	4127	497	625	3168	-	51687	2100 Variation nette de l'actif financier
...	...	...	...	...	...	112	2210 Réserves officielles de liquidités internationales
184	2249	-112	...	109	...	3447	2311 Argent liquide et dépôts bancaires
24	309	94	...	-	...	1740	2312 Dépôts dans les autres institutions
2	25	-162	...	488	-	-68	2313 Devises et dépôts étrangers
...	...	...	...	...	...	1836	2321 Crédit à la consommation
-10	26	1	...	-260	...	1574	2322 Comptes à recevoir
...	...	...	...	...	...	-599	2331 Prêts bancaires
584	16	-17	...	-521	...	1398	2332 Autres prêts
-218	-	116	...	-816	...	5740	2340 Bons du Trésor fédéraux
247	34	409	...	477	...	6141	2350 Autres effets à court terme
-8	-9	-13	...	-	...	4430	2410 Hypothèques
54	66	147	29	1843	...	3024	2421 Obligations fédérales (dont les obligations d'épargne du Canada)
...	...	...	...	...	...	...	
159	2	177	598	-145	...	1874	2422 Obligations provinciales
9	-	40	...	33	...	-107	2423 Obligations municipales
90	-2	-28	...	1816	...	1449	2424 Autres obligations canadiennes
...	...	...	...	...	...	4075	2430 Assurances-vie et rentes
-	...	...	...	-711	...	5788	2512 Créances d'entreprises privées (1)
-9	1036	398	-4	...	...	1512	2513 Créances d'entreprises publiques
90	-20	70	...	163	...	1936	2520 Actions (1)
50	-1	-	...	...	...	1028	2530 Investissements étrangers
189	396	-623	2	692	...	5357	2610 Autres éléments de l'actif financier
1300	9696	2659	...	2748	...	51687	3100 Variation nette du passif (y compris capital actions)
...	...	...	...	112	...	112	3210 Réserves officielles de liquidités internationales
...	24	...	...	...	...	3447	3311 Argent liquide et dépôts bancaires
181	-	...	...	...	...	1740	3312 Dépôts dans les autres institutions
...	...	...	...	-935	...	-68	3313 Devises et dépôts étrangers
...	...	...	...	...	...	1836	3321 Crédit à la consommation
6	-76	794	...	-33	...	1574	3322 Comptes à payer
-23	-309	-68	...	-951	...	-599	3331 Emprunts bancaires
332	-367	-5	...	91	...	1398	3332 Autres emprunts
...	5740	...	...	...	...	5740	3340 Bons du Trésor fédéraux nets de l'escompte non amorti
92	...	300	...	...	...	6141	3350 Autres effets à court terme
5	...	-	...	-	...	4430	3410 Hypothèques
...	3025	...	...	...	...	3024	3421 Obligations fédérales
-35	...	1114	...	...	...	1874	3422 Obligations provinciales
...	...	-107	...	...	...	-107	3423 Obligations municipales
266	...	-	...	...	...	1449	3424 Autres obligations canadiennes
...	-25	...	...	...	...	4075	3430 Assurances-vie et rentes
-	...	...	...	2799	...	4173	3512 Créances d'entreprises privées (1)
433	-172	82	...	...	...	1512	3513 Créances d'entreprises publiques
-	...	...	...	...	...	3551	3520 Actions (1)
...	...	...	...	1028	...	1028	3530 Investissements étrangers
43	1858	549	...	835	...	5357	3610 Autres éléments du passif
-278	-109	1517	-	278	-646	-	4000 Divergence (1900-2000)

(1) Pour les notes explicatives, voir tableau 7-1

APPENDIX B

Balancing (Q) and Time (B) Constraint Matrices

Lets define B the time constraint matrix for one individual series of length N such that N/4 is an integer.

$$B = \begin{vmatrix} 1\,1\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\ldots.0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,1\,1\,1\,1\,0\,0\,0\,0\ldots.0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,1\,1\ldots.0\,0\,0\,0\,0\,0\,0\,0 \\ \quad\vert\qquad\vert\qquad\vert\quad\ldots.\quad\vert\qquad\vert \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\ldots.1\,1\,1\,1\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\ldots.0\,0\,0\,0\,1\,1\,1\,1 \end{vmatrix}_{\frac{N}{4}\times N}$$

Note: N is the number of quarters in the series and $\frac{N}{4}$ the number of years.

$$\bar{B} = \begin{vmatrix} B & 0 & 0 & \ldots. & 0 \\ 0 & B & 0 & \ldots. & 0 \\ 0 & 0 & B & \ldots. & 0 \\ | & | & | & \ldots. & 0 \\ 0 & 0 & 0 & \ldots. & B \end{vmatrix}_{(L1\times\frac{N}{4})\times(L1\times N)}$$

where $0 = 0_{\frac{N}{4}\times N}$; $L1 = 2\times C\times S$

L1 is the number of financial flows series in X which are not total financial flow series (y_{it}) or net financial investment series (z_{jt}). These series are excluded from $\bar{B}$ because of the redundancy of these time constraints when the balancing constraints are added.

Note: B will be modified to a matrix $\bar{B}$ because of the stationarization step. See footnote 4 on page 3 .

Balancing Constraint Matrix

Let $I_k(n) = (I_{nxn} \cdots I_{nxn})$, k times k = 1,2..

$0_k = (0_{nxn} \cdots 0_{nxn})$

$\bar{Q} =$

asset	liability	Total asset and liability (category)	Net financial investment (sector)	
$I_s^{(n)}\ 0_{(s-1)xc}$	0_{sxc}	$-I_1^{(n)}\ 0_{(2xc)-1}$	0_s	
$0_1\ I_s^{(n)}\ 0_{(s-2)xc}$	0_{sxc}	$0_1 - I_1^{(n)}\ 0_{(2xc)-2}$	0_s	
$0_2\ I_s^{(n)}\ 0_{(s-3)xc}$	0_{sxc}	$0_2 - I_1^{(n)}\ 0_{(2xc)-3}$	0_s	asset category constraints
$0_{(s-1)xc}\ I_s^{(n)}$	0_{sxc}	$0_{c-1}\ -I_1^{(n)}\ 0_c$	0_s	
0_{sxc}	$I_s^{(n)}\ 0_{(s-1)xc}$	$0_c\ -I^{(n)}\ 0_{c-1}$	0_s	
0_{sxc}	$0_1\ I_s^{(n)}\ 0_{(s-2)xc}$	$0_{(c+1)}\ -I^{(n)}\ 0_{c-2}$	0_s	Liability category constraints
0_{sxc}	$0_{(s-1)xc}\ I_s$	$0_{(2xc)-1}\ -I_1^{(n)}$	0_s	
0_{sxc}	0_{sxc}	$I_1^{(n)}\ 0_{c-1}\ -I_1^{(n)}\ 0_{c-1}$	0_s	
0_{sxc}	0_{sxc}	$0_1 I_1^{(n)}\ 0_{c-2}\ 0_1 - I^{(n)}\ 0_{c-2}$	0_s	
0_{sxc}	0_{sxc}	$0_{c-1}\ I_1^{(n)}\ 0_{c-1}\ -I_1^{(n)}$	0_s	asset liability category constraints
$I_s^{(cn)}$	$-I_s^{(cn)}$	0_{2xc}	$-I_1^{(sn)}$	sector constraints

APPENDIX C

Fourier Transform of Financial Flows Table

In order to calculate the Fourier transform (for the periodogram) of each series in the financial flow matrix, we must build a matrix of Fourier coefficients.

Let F be a matrix of Fourier coefficients such that Fx_{ij} is the Fourier transform of x_{ij}, taken at the "natural frequency grid". With N observations, the natural grid has a spacing $\Delta f = 1/N$, and $fk = k/N$, $k = 0,\ldots,N/2$, when N is even

Hence:

$$F = \begin{bmatrix} \cos(0)/\sqrt{2} & \cos(0)/\sqrt{2} \ldots. & \cos(0)/\sqrt{2} \\ \cos\frac{(2\Pi)}{N} & \cos\frac{(4\Pi)}{N} \ldots. & \cos\frac{(N\Pi)}{N} \\ \cos(\Pi)/\sqrt{2} & \cos(2\Pi)/\sqrt{2} \ldots. & \cos(N\Pi)/\sqrt{2} \\ \sin\frac{(2\Pi)}{N} & \sin\frac{(4\Pi)}{N} \ldots. & \sin\frac{(N\Pi)}{N} \\ \sin 2\Pi\frac{(N-2)}{N} & \sin 2\Pi\frac{(N-2)}{N}2 \ldots. & \sin 2\Pi\frac{(N-2)}{N}N \end{bmatrix} \times \sqrt{\frac{2}{N}}$$

NxN

However since we want to calculate the Fourier transform of each series in X, then we have to build a matrix $\tilde{F}$ of Fourier coefficients

$$\tilde{F} = \begin{vmatrix} F & 0 & 0 & \cdots & 0 \\ 0 & F & 0 & \cdots & 0 \\ 0 & 0 & F & & \vdots \\ \vdots & \vdots & \vdots & & 0 \\ 0 & 0 & 0 & \cdots & F \end{vmatrix} \quad (LxN) \times (LxN)$$

where L is the number of series in X.

APPENDIX D

Stationarisation of Each Individual Financial Flows Series

The stationarization involves only one series at a time. So, when the first difference is taken and then the result is divided by the annual total, it affects separately each identity submatrix in Q. In step adjustment procedure, the effect on B is more important since B is not built with identity submatrices.

The two types of relations we start with for a particular financial flows series and within the category and sector constraints are of the form

$$I_{NxN} \times x_{ij} = x_{ij}$$

and $$0_{NxN} \times x_{ij} = 0_{Nx1}$$

where $x_{ij} = (x_{ijt})_{Nx1}$ $\qquad t=1,\ldots,N$

In the step adjustment procedure, x_{ij} has been modified to a vector Δx_{ij} such that

$$\Delta x_{ij} = \begin{bmatrix} (x_{ij2} - x_{ij1})/x^{1}_{ij} \\ (x_{ij3} - x_{ij2})/x^{1}_{ij} \\ (x_{ij4} - x_{ij3})/x^{1}_{ij} \\ (x_{ij5} - x_{ij4})/x^{2}_{ij} \\ \vdots \\ (x_{ijk} - x_{ij,k-1})/x^{\upsilon}_{ij} \\ (x_{ij,k+1} - x_{ijk})/x^{\upsilon}_{ij} \\ (x_{ij,k+2} - x_{ij,k+1})/x^{\upsilon}_{ij} \\ (x_{ij,k+3} - x_{ij,k+2})/x^{\upsilon}_{ij} \\ (x_{ij,k+4} - x_{ij,k+3})/x^{\upsilon+1}_{ij} \\ \vdots \\ (x_{ijN} - x_{ij,N-1})/x^{N/4}_{ij} \end{bmatrix}$$

where $\upsilon = (k + 3)/4$

= number of years corresponding to the involved quarters.

When we apply a first difference to $I_{NxN} \times x_{ij}$, we obtain

$$(1-B)[I_{NxN} \times x_{ij}] = I_{(N-1)x(N-1)} \times \begin{bmatrix} (x_{ij2} - x_{ij1}) \\ \vdots \\ (x_{ijN} - x_{ij,N-1}) \end{bmatrix}$$

Now, $I_{(N-1)x(N-1)}$ has to be modified to take into account the division by the yearly totals and also to make sure that the category and sector constraints still hold. If the yearly totals were a constant for the whole series, $I_{(N-1)x(N-1)}$ should not have to be modified, since the constraints would hold. Since it is not the case, $I_{(N-1)x(N-1)}$ is changed to get rid of the denominators and become

$$A_{ij} = \begin{bmatrix} x^1_{ij} & & & & & \\ & x^1_{ij} & & 0 & & \\ & & x^1_{ij} & & & \\ & & & x^2_{ij} & & \\ & 0 & & & \ddots & \\ & & & & & x^{N/4}_{ij} \end{bmatrix}_{(N-1)x(N-1)}$$

We then replace each identity matrix $I_{N,N}$ in the original categoryu and sector constraint matrix by the corresponding A_{ij} matrix for the x_{ij} financial flow series.

In the relation $0_{NxN} \times x_{ij}$, the matrices 0_{NxN} remains unchanged.

APPENDIX E

Standardisation of Each Individual

Financial Flows Series

The standardization of each individual modified for stationarity financial flows series Δx_{ij} will not affect the constraints matrix, but that will change the right hand portion of the contraints equation.

$$\text{Let } \mu_{ij} = \sum_{t=1}^{N-1} \Delta x_{ijt}/(N-1) \qquad \forall i,j$$

$$s_{ij} = \left[\sum_{t=1}^{N-1} (\Delta X_{ijt} - \mu_{ij})^2/(N-1)\right]^{1/2} \qquad \forall i,j$$

$$\mu = \begin{bmatrix} \mu_{11} \\ \mu_{12} \\ \cdot \\ \cdot \\ \cdot \\ \mu_{2C,S} \end{bmatrix}$$

$$SD = \begin{bmatrix} s_{11} & 0 & .. & 0 \\ 0 & s_{12} & .. & 0 \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ 0 & 0 & .. & s_{2C,S} \end{bmatrix}$$

$$\Delta X^s = (\Delta X - \mu) \times SD^{-1}$$

where s means standardization.

The stationarized constraints equation before standardization is

$$1.\quad \begin{bmatrix} B \\ C \end{bmatrix} \times \Delta X = \begin{bmatrix} \Delta Y \\ 0 \end{bmatrix}$$

or 2. $V \Delta X = W 1$

where $V = \begin{bmatrix} B \\ C \end{bmatrix}$

$\Delta Y = \begin{bmatrix} y_1 \\ . \\ . \\ . \\ ya \end{bmatrix}$ is the vector of length a = L1x((N/4)-1) of annual first difference divided by the current year value

0 is a nul vector of length b= Ncx(N-1)

$$W = \begin{bmatrix} y_1 & 0 & 0 & \ldots & 0 \\ 0 & y_2 & 0 & \ldots & 0 \\ 0 & 0 & y_3 & \ldots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \ldots\ ya\ \ldots & 0 \\ 0 & 0 & 0 & \ldots\ 0\ \ldots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix} \quad (a+b) \times (Lx(N-1))$$

1 is a vector of 1 value of length Lx(N-1)

N_c is the number of balancing constraints for a given quarter

L is the number of series in the financial flows table.

The standardization involves changing ΔX to ΔX^s such that

$$\Delta X^s = (\Delta X - \mu) \times SD^{-1}$$

where μ is

SD is a diagonal matrix of standard deviations

The left part of the stationarized constraints equation 2 becomes

$$\begin{aligned} V\ \Delta X^s &= V \left[(\Delta X - \mu) \times SD^{-1} \right]' \\ &= V \left[\Delta X' \times SD^{-1} \right]' - V \left[\mu' \times SD^{-1} \right]' \\ &= V\ SD^{-1'}\ \Delta X - V\ SD^{-1'}\ \mu \\ &= V\ SD^{-1}\ \Delta X - V\ SD^{-1}\ \mu \end{aligned}$$

But $W \left[1' \times SD^{-1} \right]' = W\ SD^{-1}\ 1$

$= W\ SD^{-1}\ 1$

Then equation 2 becomes

$$3.\quad V\ \Delta X^{s\prime} = W\ SD^{-1}\ 1 - V\ SD^{-1}\ \mu$$

Equation 3 is the standardized, stationarized constraints equation

APPENDIX F

Minimization

Let's define $V = \begin{bmatrix} \tilde{A} \\ \tilde{\eta} \end{bmatrix}$ the balancing constraints matrix for the stationarized and standardized financial flow vector ΔX^S

and $T = W \quad SD^{-1}1 - V\, SD^{-1}\, \mu$

We want then to minimize

$$(\Delta X^{Bs} - \Delta X^s)'\ \tilde{F}'\tilde{F}\ (\Delta X^{Bs} - \Delta X^s)$$

subject to $V\Delta x^{Bs} = T$

where Δx^{Bs} is the balanced stationarized and standardized financial flow vector.

In Lagrangian's notation, we wish to minimize

$$M : (\Delta X^{Bs} - \Delta X^s)'\ \tilde{F}'\tilde{F}\ (\Delta X^{Bs} - \Delta X^s) + \lambda'\ (V\Delta X^{Bs} - T) = 0$$

can be rewritten in the following way:

$$M : (\Delta X^{Bs} - \Delta X^s)'\ (\tilde{F}'\tilde{F})(\Delta X^{Bs} - \Delta X^s) + \lambda' \left[V(\Delta X^{Bs} - \Delta X^s) + V\Delta X^s - T\right] = 0$$

By differentiating according to $(\Delta x^{Bs} - \Delta X^s)'$ and λ', we obtain

$$2\,\tilde{F}'\,\tilde{F}(\Delta X^{Bs} - \Delta X^s) + \lambda' V = 0$$

$$V\Delta X^{Bs} = T$$

or $$\begin{bmatrix} 2\,\tilde{F}'\,\tilde{F} & V' \\ V & 0 \end{bmatrix} \times \begin{bmatrix} \Delta X^{Bs} \\ \lambda \end{bmatrix} = \begin{bmatrix} 2\,\tilde{F}'\,\tilde{F}\Delta X^s \\ T \end{bmatrix}$$

The solution of this system of equations involves inverting

$$G = \begin{bmatrix} 2\,\tilde{F}'\,\tilde{F} & V' \\ V & 0 \end{bmatrix}$$

Note: $\tilde{F}'\,\tilde{F}$ is the identity matrix.